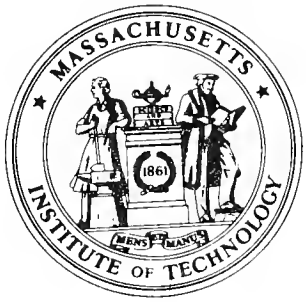

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On the Trail of Conventional Beliefs About the Transfer Problem*

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ON THE TRAIL OF CONVENTIONAL BELIEFS ABOUT THE TRANSFER PROBLEM

Paul A. Samuelson*

I. Introduction

When Germany was made to pay war reparations at Versailles, economists fell into dispute on the question of whether a unilateral transfer will, aside from the primary burden of the payment itself, also cause a secondary burden as a result of a presumed induced deterioration of the terms of trade of the paying country. In this famous transfer problem debate, Keynes, Pigou, Taussig, Robertson, and many others upheld this orthodox view of presumed secondary deterioration. Ohlin, pointing out that income effects had been neglected, asserted that no such clear-cut presumption was possible; and Viner demonstrated that the classical writers were by no means unanimous in holding to the orthodox position.[1]

Analytically the discussion remained confused, since models involving effective demand and financial considerations were rarely carefully separated from those involving pure barter. (In connection with barter models, even the great Marshall erred by shifting the paying country's offer curve while leaving that of the receiving country intact.) Empirically, the situation also remained confused, since it was not even clear whether Germany's reparations equalled the unilateral investments made to her from abroad.

Not until the 1930's did Pigou[2] clarify the barter aspects by appeal to an exact Jevons model of exchange. This enabled one to see how much the orthodox result depended upon particular assumptions made about transport costs and

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impediments in international trade. In two rather exhaustive articles,[3] I concluded that, in the absence of transport costs or impediments, the orthodox presumption lacked basis (thus, in a sense, awarding the palm to Ohlin as against the pre-General Theory Keynes). With transport costs and tariff impediments, the outcome was shown to be very complex indeed since once we isolated the crucial income-propensities upon which the result depended, clear-cut presumptions became difficult.

There the matter stood until recently Professor Ronald Jones[4] provided a beautifully simple argument that demonstrates an anti-orthodox (and, hence, partially anti-Ohlin) presumption even in the purest model involving zero transport costs and tariffs. Jones shows that random differences in tastes, which are independent of random differences in comparative advantage, result in countries' tending to import goods that are peculiarly taste-appreciated by them. (If I am a drunkard and you are a fop, I am more likely to export cloth and import rye than vice versa.) In consequence of Jones's hypothesis, my test criteria deduce that the receiving country is likely to have a deterioration of its terms of trade, giving to the paying country a secondary-blessing rather than the secondary-burden of the orthodox school or the zero burden of the Ohlin school.

I applaud the Jones result. Yet, even though I know better, I often find myself falling into the orthodox presumption. Why is this? Is it forgetfulness? Is it stupidity? Or is it perhaps that I, along with Keynes and Taussig, have implicitly in mind a tempting model in which the orthodox result is legitimately implied? The present paper explores the affirmative answer to the last question and hopes to throw light on the reasons that earlier writers fell into the orthodox view.

II. Partial Equilibrium Models

Implicitly, economists tend to use partial equilibrium models and to combine financial analysis with real. Less systematically, they tend to use simple Ricardian constant costs. Put all this together and you will not be surprised at the orthodox thesis. Moreover, by some sophisticated specifications, these top-of-the-mind notions can be made part of an exact general equilibrium model that combines Marshall, Ricardo, and, for that matter, Hume. However, I confine myself here to the transfer problem, leaving for publication elsewhere[5] the rigorous general equilibrium model.

Begin with Figure 1 which provides an interesting variant on the familiar back-to-back diagrams of Cournot, Barone, Bickerdike, Joan Robinson, Haberler and many others.[6] In (a) we have the usual supply and demand curves for wheat in America expressed in dollars. In (c) we have similar curves for wheat but expressed for England in terms of the pound. In (b) the foreign exchange rate, R, giving the dollar costs of 1 £ (e.g. \$3/£) is denoted by the slope of the OR ray. (The exchange rate giving the pound cost of \$1, r = 1/R, is denoted by the slope of the ray referred to the vertical axis, thus preserving symmetry between countries.)

At each exchange rate, R, there is determined an equilibrium dollar wheat price, P_1, also a pound wheat price, p_1, and, finally, the physical (algebraic) export of wheat from America to England. We can write the algebraic export functions as the difference between supply and demand in the respective countries, namely $E_1(P_1) = S_1(P_1)-D_1(P_1)$ and $e_1(p_1) = s_1(p_1)-d_1(p_1)$ with E_1' and e_1' posited to be positive as a condition of stability. (This says the supply schedule, if negative, must be more vertical than the demand curve, in the usual Walrasian manner.)

Competitive arbitrage requires the prices in two markets be equal when expressed in common currency units. Hence,

$$(1) \qquad P_1 = R\,p_1$$

Finally, equilibrium is determined by the equality of export of wheat from one country with import of wheat in the other; namely, by solving

$$(2) \qquad -e_1(p_1) = E_1(RP_1)$$

to get p_1 as a declining function of R. I.e., appreciating the pound will lower the price of wheat in London. By symmetry, depreciating the dollar relative to the pound (i.e. raising R) will tend to raise the dollar price of wheat, making the American dollar P_1 an increasing function of the foreign exchange rate.

Figure 1 shows all this. As the pound depreciates, OR pivots clockwise. As it does so, American wheat exports decline. When the exchange ray shifts to OR' in 1(b), all trade ceases.

Now suppose that wheat were the only good. Balance-of-payments equilibrium requires an equivalence of aggregate value of exports to aggregate value of imports. If wheat is the only good, this can be realized only when R depreciates to the OR' level of zero wheat export. This equilibrium R^* is given by the solution of

$$(3) \qquad E_i(RP_i) = -e_i(p_i) \quad , \; i=1$$

$$\sum_{i=1}^{1} p_i e_i(p_i) = p_1 e_1(p_1) = 0$$

We thus have, in addition to the balance-of-payments equation, an export equation for every good, namely in our case of the single good wheat (2) above. Hence, we do always have n + 1 equations to determine the n p's and the equilibrium R^*. From p's, we get P's by (1).

III. Exchange Depreciation for the Paying Country

What is the moral? What implication is there for the transfer problem? Suppose America is now to make a unilateral payment to England. How will the previous 1-good autarky equilibrium at OR' be disturbed? Obviously, OR' must pivot upward (counterclockwise) toward OR. For such a depreciation of the dollar is the only way[7] that an American trade surplus can be generated.

Thus, the Keynes intuition that the paying country's exchange rate depreciates is vindicated in this primitive model. But this tells us nothing about any secondary movement induced in the terms of trade. Indeed, with wheat the only good there is no price ratio, P_1/P_2, for America to compute as a measure of the (so-called Taussigian net) terms of trade.

IV. Constancy of Some Marginal Utility

Before introducing a second good, we must notice that when America produces more wheat than she consumes, this must be financed. Since there is no investment or disinvestment in our model, how can the value of net national product fall short of the value of net national income received from wheat production? Evidently this value of the export surplus is financed by American government taxation of earned income to pay for the reparations. (In England, the reverse problem is easily resolved. English product consumed exceeds income earned domestically by the amount of government subsidy. But how does the English government finance this subsidy? From its "reparation" receipts.)

The reparation is, in effect, paid in wheat. One might even have the American tax and the English subsidy paid in terms of wheat. Everything would then seem to be in terms of wheat, the only good named. If wheat were truly

the only economic good, what is the meaning of supply and demand curves? What is the American dollar-supply-and-demand or P_1? What is English pound supply-and-demand or p_1? On reflection, we realize that "money" must be a Marshallian euphemism for some domestic good or a host of such goods and factors.

Let us postulate that wheat is producible in each country by labor applied to fixed wheat land. The law of diminishing returns shows up as the rising supply functions. More precisely, suppose we use labor hours as our numeraire, so that wage rates, W in America and w in England, are each set equal to unity. Then our functions, $S_1(\cdot)$, $D_1(\cdot)$, $E_1(\cdot)$, are really functions of P_1/W. Likewise, we write $s_1(p_1/w)$, $d_1(p_1/w)$, $e_1(p_1/w)$.

But there is still a snag. When governments tax us or subsidize us, particularly if these fiscal variables were payable in wheat rather than in wage-units of income, that can usually be expected to distort and alter the Marshallian demand functions. How does a conscientious Marshallian get rid of this distortion and complication? He can do so if he stipulates that labor has strictly constant marginal disutility (or that leisure, regarded as "money," has strictly constant marginal utility). In this singular model, all consumer-surplus and producer-surplus concepts are valid. And, what is important to an Ohlin skeptic, all income effects are exhausted on labor (or leisure), which unlike wheat and other traded goods, are purely domestic goods.

One last caveat if my Marshallian model is to be rigorous. As one moves up the supply curve, the absolute level of rent rises. Property's relative share is given by the ratio of two areas: the producer-surplus "triangle" between the supply curve and the rectangle formed by each supply point all divided by the rectangle's whole area. However, we need not worry if landlords have different wheat-leisure preferences than workers or worry about intra-class taste differences. Provided everybody values leisure at constant marginal

utility and is not satiated with it, each person's independent demands for wheat, or cloth, or steel will be simple added to form non-shifting aggregate demand functions, $D_i(P_i/W)$ and $d_i(p_i/w)$. In order that the supply functions for the different goods be independent, I postulate the Ricardo-Viner case where labor is the only transferable input between industries, all wheat-lands and wool-lands being specialized to single industries and earning simple Ricardian residual rents. This can justify the independence and invariance of the $S_i(P_i/W)$ and $s_i(p_i/w)$ functions.

V. Many Goods

Now in Figure 2 we add a second good along with wheat, say wool or cloth. Now at an exchange rate like OR, each country has an export advantage in one of the goods, America in wheat and England in cloth. Before reparations the common OR ray in 1(b) and 2(b) provides equilibrium R^* because the export and import rectangles in each country cancel out to zero or balance-of-payments equilibrium.

If we try a higher R, England runs a deficit. If we try a lower R, the dollar (and r) appreciates, and America runs a trade deficit. The R^* equilibrium is unique and stable.[8]

Mathematically, our equilibrium is defined by

$$(4) \qquad E_i(RP_i) + e_i(p_i) = 0 \quad , \quad (i = 1, 2, \ldots, n)$$

defining

$$p_i = p_i[R] \quad , \; p_i' < 0$$

$$P_i = P_i[R] \quad , \; P_i' > 0$$

and

$$b[R] = \sum_1^n p_i e_i(p_i) = \sum_1^n p_i e_i(p_i[R])$$

$$= \sum_1^n p_i e_i[R] = 0 \quad , \text{ where } e_i'[R] < 0$$

All the ink spilled over so-called Marshall-Lerner stability conditions [for which see Haberler's 1949 Kyklos citation of works of Hirschman, A. C. Brown, Metzler, and others beyond those I had already cited] merely produces breakdowns of the terms involved in $b'[R]$. It suffices for me to note that $b' < 0$ is not inevitable even through probable in many specifications.

Now return to the transfer problem. If America now pays an indemnity to England, how must the two-good OR ray of equilibrium rotate to produce an American export surplus? Clearly the dollar must depreciate and OR must move clockwise (as to OR" in Figure 2), thereby increasing our wheat export and diminishing our cloth imports. In symbols, if America pays Europe a unilateral transfer of $t, we replace (5)'s balance-of-payment condition by $-t$ rather than zero on the right-hand side, namely

$$(6) \qquad b[R] = -t$$

and, provided $b' < 0$, the stable case, we confirm for any number of goods the earlier theorem, namely:

Theorem: The exchange rate of the paying country is depreciated by a unilateral transfer.

VI. Deteriorating Gross-Barter Terms of Trade

Because currency depreciation increases every physical export algebraically (i.e. cuts down on any physical import), the paying country is "hurt" by the transfer. This shows that, regardless of what happens secondarily to the (Taussigian net) terms of trade, the 1936 Leontief phenomenon--of an alleged possibility that a transfer can have secondary effects so much opposed to the primary burden as to lead to a utility gain to the payer--definitely cannot happen in our model.

For a two-good model Taussig's "gross barter terms of trade," the ratio of the paying country's physical imports to its physical exports, $e_2[R]/-e_1[R]$ in the wheat-cloth case, must definitely deteriorate. This is because the numerator declines with R and the denominator grows with R. Of course there is nothing very surprising about this result, since one does expect there to be a primary burden.

When we face more than one export and more than one import, and when goods can pass from one category into the other, an index number problem arises for any definition of the terms of trade, gross or net. Our 2-good result concerning the deterioration of gross-barter terms of trade can, however, be generalized as follows.

In view of the fact that every algebraic export from England is reduced by the transfer receipt, if we can divide the n goods into m American imports $e_i[R]$ and n-m American exports $-e_j[R]$, then for any fixed positive weights $(k_1,k_2,\ldots,k_m,k_{m+1},\ldots,k_n)$, the following definition of the gross-barter terms of trade, $\sum_1^n k_i e_i[R] e_j[R] \sum_{m+1}^n -k_j e_j[R]$ will definitely fall, just as in the case of the 2-goods example.

VII. Likely, But Not Inevitable, Secondary Burden

To explain impressionistic views of the past, we have come far enough. In the present partial-equilibrium model (which elsewhere I call the B-R-H model because Bickerdike, Joan Robinson, and Haberler have perhaps written most about it), the transfer does depreciate the currency of the paying country.[9] And for the majority of economists a deterioration of the exchange rate is practically identified with a deterioration of the terms of trade. Indeed some economists even go so far in their thinking as to believe that it is the lowering of the terms of trade which brings about the correction in

the trade balance incident to a currency depreciation. This too-facile identification of the terms of trade with the foreign exchange rate has many roots. For one thing, as already mentioned, there is the common confusing together of barter and financial models. Thus an expression like the Marshall-Lerner elasticity criterion for stability is used interchangeably for (1) a barter model like that involved in Marshall's 1879 Pure Theory of Foreign and Domestic Commerce or the Appendix to his 1923 Money, Credit and Commerce where reciprocal offer curves cross and (2) a money model like the B-R-H one now under discussion. (Of course, this model is itself not really a financial model but is also a special kind of barter model, as my version of it makes clear for perhaps the first time.) In the typical two-good barter model of J. S. Mill, Marshall (1879), Edgeworth, and others, stability is restored only by changes in the price ratio of the traded goods or the [net] terms of trade. But such reasoning is not really valid for the model here under discussion.

For many years after Bickerdike, in 1920, first published such a model, economists took it for granted that exchange depreciation causes a nation's export prices to fall relative to its import prices. Even in the gold-standard case, equilibrium is loosely said to be restored as prices of the deficit country fall relative to prices of the surplus country. (In the yet-to-be-published companion paper to this one, I analyze the flaw involved in such wording.) Actually, however, as we have seen, and as our diagrams will confirm, when R rises in virtue of depreciation of the dollar, prices of both export and import goods fall in dollar terms; and prices of both British imports and exports rise in pound terms. And so it is gratuitous to jump to the conclusion that the prices of exports of the depreciating country drop relative to those of its imports.

Yet it was not until the late 1930's that Joan Robinson[10] pointed this out in the literature--much to the surprise of specialists in international

trade theory. When they first became aware of the Robinson curiosum, they thought that perhaps her effect was brought about only in cases where the equilibrium was unstable by virtue of reversal of the Marshall-Lerner stability criterion. But it was easy to provide examples of perfectly stable equilibria where the terms of trade actually turned in favor of the depreciating country, and yet depreciation did restore the disturbed equilibrium.

We can state then that it is quite possible for the secondary effects of the transfer payment to go either way: the net terms of trade of the paying country could improve or they could deteriorate as a result of the transfer.

To see this, consider a 2-good case of say wheat and cloth. It is easy to construct along Robinson lines an example of a stable system and in which P_1/P_2 rises as a result of the transfer rather than falls as the orthodox view would hold. Where economists went wrong in denying this possibility was in comparing prices across countries--comparing a P with a p, as for example P_1 with p_1. It is true that the price of cloth has risen in dollars and fallen in pounds. But, since such a ratio is nothing but the exchange rate in a free market, what else could it do when R changes? Such a ratio has nothing to do with a properly computed measure of the terms of trade, which should involve a comparison of export prices with import prices in the same market, whether it be the dollar or pound market. Thus, whatever P_1/P_2 does, so must p_1/p_2 do, since they are the same thing. (I.e., cancelling out R from numerator and denominator of the former will give you the latter, no matter what is the new exchange rate R.) When index numbers of prices become involved, and also certain impediments and costs of trade, one can become confused on these fundamentals. Such confusions are common in connection with historic discussions of purchasing-power-parity, as I have discussed elsewhere;[11] and I fear that sometimes Keynes added to the confusion on this matter.

Although the Robinson curiosum shows that no unambiguous answer is possible concerning the secondary burden of a transfer, economists have tended to think that this phenomenon is abnormal in the sense of being unusual. Since we are discussing what is after all a rather idealized model, I am not sure that sense can be made of a statement that one result is "empirically more realistic" than another. But in any case, economists have often supplied conditions which they hoped were sufficient to rule out the Robinson curiosum. Broadly speaking, they sought such conditions in extreme elasticity of postulated supply. In my present concern to try to explain why older economists fell naturally into the orthodox view, I shall now present a rather extreme case of elastic supply--namely, the classical model of Ricardian constant costs. For, within that model, we shall see that the Marshallian partial-equilibrium approach does lead to deterioration of the terms of trade along the lines of the orthodox presumption.

VIII. Combining Marshallian Curves and Ricardian Comparative Advantage

Since my unpublished companion paper elaborates upon the present subject and since my 1962 paper just cited presented a complete account of the production side of the many-good Ricardian model, I shall be brief here and omit diagrams.

Now labor is assumed to be the only factor, $A_1, A_2, A_3, \ldots$ units of American labor being required to produce a unit of goods $1,2,3,\ldots$. To produce a unit of those same goods in England requires $a_1, a_2, a_3, \ldots$ units of English labor. Recall that W is the dollar wage rate for American labor and that w is the pound wage rate for English labor. In good classical fashion we rule out migration between countries. As before, the exchange rate giving the cost of 1 pound in terms of dollars is R, with $r = 1/R$ being the number of pounds per dollar.

How is supply affected in Figures 1 and 2? Merely by having the supply curves all become horizontal, in consequence of the Ricardian assumption of constant costs. I.e., with labor the only factor and lands ignorable, diminishing returns has no scope. Elsewhere I spell out the minor variations needed in the B-R-H model when it is made Ricardian. (This involves merely introducing some inequalities into equations (4) above.) However, we can dispense with that examination here, since on reflection we notice that the way relative prices change is completely predictable from the movement of the exchange rate alone.

For it is an easy theorem in the constant-cost case that depreciating the dollar must reduce all prices of the goods America exports in comparison with those she imports; and it will leave intact the price ratios of all goods that a country continues to produce.

Where are these price ratios to be measured? In England or in America? It does not matter; in a perfect market the price ratios in one country are identical with those in another, regardless of the production origin of the goods in question.

Specifically, for any two goods that America continues to produce, the price ratio is given by A_i/A_j. For any two goods that England continues to produce, the price ratio anywhere in the world is given by a_i/a_j. The only interesting case then is the comparison, in any one place, of the price ratio of an American export to a European export as R rises. Let us consider the net terms of trade as England sees them between cloth and wheat as measured by p_2/p_1 or P_2/P_1. Using the latter dollar version, we calculate it as the European labor cost of cloth, a_2, translated from pounds into dollars by the exchange rate R, all divided by American labor cost (in dollars) of wheat. Hence, so long as each country's cost for its indicated export is the lowest cost anywhere in the world, the expression for England's terms of trade becomes

$$p_2/p_1 = a_2wR/A_1W$$

This is seen to be linear in the exchange rate and hence grows whenever R grows (up to the point where England is priced out of the market and America's A_2/A_1 takes over or down to the point where America is priced out of the market and a_2/a_1 prevails). Of course a change in the exchange rate might be offset by a wage rise in the depreciating country and a wage fall in the appreciating country. But the real transfer will not be possible unless the wR/W factor in the above expression does in fact rise and that is all we need for the argument. Incidentally, as remarked before, in a footnote, although my discussion is couched primarily in terms of flexible exchange rates, it applies fully to the gold standard case of fixed parities: in that case, R stays constant, but, as a result of specie flows or managed money, the called-for percentage changes all take place in the wage ratio w/W.

Let us take stock to see what the Ricardian case has added to the intuitive expectations of literary economists concerning the transfer problem. What constant cost has done is to rule out the Robinson curiosum.

Now any exchange depreciation induced by a unilateral payment must (save in the limiting and frequent case, beloved by Frank Graham, of "limbo" where one country is both producing and importing a good in proportions determined by reciprocal demand and where secondary burdens are nil) create a secondary burden on the paying country in the form of a deterioration of its export prices relative to its import prices. Q.E.D.

My task is completed. I have shown that any economist who reasoned in the back of his mind in terms of simplified partial-equilibrium industries, foreign exchange equilibrium, and comparative cost, could be forgiven from falling into the orthodox presumption in connection with the transfer problem. For that presumption is a valid theorem in terms of the B-R-H model, or at least in my B-R-H-S version of it.

IV. Comparative-static Stability in the Large

He who reads and runs can now run. But there are a couple of loose threads that I would like to tie up. First, as mentioned before, there is the possibility of a locally-unstable equilibrium. (And I must stress that the Ricardian horizontal supply does not, for all its infinite elasticity, rule that out--even if it perhaps makes instability less likely and does negate the Robinson curiosum.) As far as both dynamics and comparative statics are concerned, such local instability does not--repeat, not--change the conclusion that a transfer will end up deteriorating the exchange rate of the payer. This strong conclusion, which might be dignified as being an extension of the "correspondence principle,"[12] seems to have been overlooked and misunderstood in the literature. Second, there remains the minor problem of relating this defense of the orthodox position to my earlier 1952 and 1954 exhaustive examination and near-rejection of that position.

Figure 3 illustrates a case of multiple equilibrium. Between the two stable equilibria, A and C, there is the unstable equilibrium B. Now along comes an indemnity of t paid to Britain (say, in pounds). This shifts our plot of -b[R] to the right. Or what is the same thing, we find the new equilibrium in Figure 3 by looking for the intersection of the curve, not as before with the vertical azis, but with the vertical line t distance to the left of the axis. And again we have three equilibria, stable a and c and unstable b.

This accords with the usual discussions. Writers say, "See how the stable A has shifted upward, to a, as the receiving country's currency appreciates. And see how the stable C shifts upward to c. All this is as one should expect: according to the simplest correspondence principle, when an equilibrium is dynamically stable, its comparative statics behaves normally.

But see how the unstable B shifts perversely to b. This too is in accordance with the correspondence principle which relates perverse dynamics to perverse comparative statics. And now we have room for paradox. If the world begins at the unstable B, America could better her terms of trade by forcing a reparation payment on England!"

Some of this argument is correct. But some is simply wrong. It is true that a system which starts at either stable equilibrium will move upward to the indicated new equilibrium--with all the effects indicated in the present paper. But it is simply not true that starting the system at the unstable B equilibrium, and then disturbing it by a payment to England, will cause R to fall--either in the end or at any time during the transitional process.

Let us see what must actually happen. The instant the payment begins England runs a surplus in the foreign exchange markets (i.e., in the first instant we move due west of B). In the second instant, this bids up the price of the pound, R. (This is in accord with the dynamic adjustment equation which says that dR/d(time) has the algebraic sign of "excess-demand" for pounds.) So in the transition R rises just as in the stable cases. In those stable cases, however, the transition ends when R has risen into the new, nearby, stable equilibrium. What happens in this unstable case? There is no new, nearby, stable equilibrium, so the transition process just goes on and on. R continues to rise. Continues to rise forever? No, it cannot rise above the highest stable equilibrium c. And so it rises until that equilibrium is reached. In short, we always have a comparative-static rise in R with or without instability! So we can say that "in the large", as contrasted to local unstable and irrelevant branches of the b[R]+t=0 relation, the final observed dR/d(transfer) movements have the same positive sign as in the stable case. No matter how many stable and unstable equilibria, every initial equilibrium is displaced into a higher stable equilibrium. Q.E.D.

How this sweeping correspondence result must be modified when the problem is more than two dimensional must remain an open question. However, our result does have some policy implications: if momentarily trapped in an unstable equilibrium, and it becomes a nice question as to how that could have come about, a country can benefit by using its monopoly power and restricting goods in international trade so as to force the world into a stable equilibrium more agreeable to the country in question. But it cannot do so by throwing its bread on the waters in the form of gifts and transfers. (Also one shudders at the prospect of two von Neumann countries jockeying for position between stable equilibria more agreeable to one than to the other--an indeterminate problem in bilateral monopoly.)

Figure 3 brings out another facet of the policy problem. From Versailles on, Keynes warned that Germany might not have the capacity to pay the imposed reparations. (Some have dismissed this as Francophobia; and in the Weimar republic, Schumpeter made himself rather unpopular by saying that of course Germany could pay the reparations if she set her will to it.) Whether Keynes was right in his impressionistic econometrics back in 1919, Figure 3 shows that if the vertical line is moved too far westward, there might be no possible equilibrium. Making Germany pay so large an indemnity would result merely in an endless depreciation of the mark and galloping inflation inside the country. (Whether the 1920-23 German hyperinflation had much to do with reparations is not at all clear.)

Finally, there remains the task of reconciling the present strong affirmation of the orthodox position with the earlier skepticism concerning it.

X. Reconciliation with 1952-4 Results

Of my two cited Economic Journal articles on the transfer problem, it was the first 1952 one which largely assumed no transport costs and no domestic goods. And it was that article which (the new Jones result aside) demonstrated that Ohlin agnosticism was warranted in criticism of the relative income propensities for the two goods in the two countries; and with transport costs ignorable and demand not at all localized, the principle of insufficient reason suggested that any result was equally likely. How does the present model fit in?

Both wheat and cloth are certainly freely transportable. However, leisure in America and leisure in England are not internationally-transferable items. They are domestic goods par excellence, with so to speak infinite transport costs as the result of the usual Ricardian assumptions that factors are immobile in international trade. Recall that leisure and labor are different names, or different aspects, of the same thing. So at first blush, one is inclined to say that the present paper takes us out of the agnosticism of my first paper and into the morass of my second 1954 paper, with its many possible patterns of transport costs.[13] But that would seem too-sweeping a conclusion. What use would my first 1952 model be if the existence of any immobile factors were considered to render it inapplicable? It, after all, did allow explicitly for transformation tradeoffs between the two goods, with labor and land thus being understood to be in the background; and nothing could be less mobile than land. So it cannot be the mere presence of localized labor that frees the present inquiry from the conclusions of my first paper.

At a slightly deeper level, it might be argued that "leisure" never entered into my 1952 paper at all. Labor did, but the supply of labor like the supply of every factor was there implicitly taken as fixed. So there

was no need, explicitly or implicitly, to consider leisure. In other words, it is the variability of the supply of labor or of some domestic good that seems to be involved here. This suggests that the present model falls into the category of what was described on p. 302 of the 1952 article (p. 1099 of my Collected Scientific Papers, II) as follows: "To my knowledge the only logically air-tight successful defense of the orthodox view is that given by Viner [Studies, p. 348-9], in which he explicitly introduces into the problem transport costs great enough to make international trade prohibitive for some 'domestic commodities.' Naturally a high percentage of our income is spent on such commodities."

Hence it does become likely that my test criterion involving income propensities on food and cloth of payer and receiver will indicate the orthodox presumption. (And as is said in a footnote on the next page: "Viner's successful vindication of the orthodox presumption was possible because of his (quite realistic) introduction of an element of asymmetry into the problem: his domestic good is made (infinitely) substitutable for the region's export good production and not at all substitutable for the import good production.") Simple mathematical analysis can show that the Ricardian case analyzed here is indeed of this general class involving asymmetric relations between a country's exports and its domestic goods or factors in variable supply.

To show this I shall make use of an Edgeworthian device pioneered by James Meade, the trade-indifference contours. (In the cited Caves-Johnson AEA reproduction of my 1952-4, I added a 1966 postscript suggesting that my case of variable production could be reduced to "box-diagram" format by the use of the now-familiar Meade device. But for the present purpose I have to go beyond the usual form of the Meade device and optimize also with respect to labor supply in the background. I.e., for prescribed levels of $-E_1$ and

$-E_2$, I maximize America's utility from wheat consumed, cloth consumed, and leisure enjoyed, subject of course to the Ricardian labor-cost constraints. This gives me in the end "contours of trade indifference." Figure 4 illustrates these for America and Europe. The European contours are shown as broken lines and are to read from right to left and upside down in the usual box-diagram fashion.

Actually, and this will come as no surprise to students of Kuhn-Tucker nonlinear programming or of Edgeworthian trade theory, these contours will now be parallel straight lines in a large part of the field (i.e., where a country is producing something of goods), with slope equal to A_2/A_1 or a_2/a_1 as the case may be. Beyond the boundaries of this field, the differences in comparative cost will introduce discernible asymmetries in Figure 4's income propensities or Engel's curves. Thus, suppose we begin at a pre-reparation point where each country is specializing on the good in which it has a comparative advantage. And let us evaluate the crucial income propensities there to see if we do find that they differ in a systematic fashion predictable from comparative-cost theory alone. The answer is, Yes. An international income change (due to reparation or anything else) alters the amount exchanged of a country's export good alone. In that case transferring abstract purchasing power from America to England will cause the price of England's export good, cloth, to be bid up (by her) and will cause the price of America's export good, wheat, to be bid down (by the drop in America's income). Thus the orthodox presumption is indeed triumphantly established. Q.E.D.

A last word. One of the triumphs of the 1952 paper was the demonstration that, for stable systems, no matter what the complexity of conditions in the background, the simple food-cloth income propensities were determinative. What then do they tell us about the present case? Because of the assumption that leisure-labor has constant marginal utility, all consumption-income effects

for cloth and wheat are zero in both countries. So my 1952 criterion becomes 0/0, an indeterminate number which tells us nothing but which is consistent with orthodox deterioration proved by other methods.

Mathematically, I shall reduce the behavior equations of the problem to the solution of a certain maximum problem for each country. We have seen that the independent demand curves of the different citizens can be horizontally added to get the market demand curves. These individual demand curves are, by virtue of our assumption of constant marginal utility of leisure-labor, identical to marginal utilities for each citizen. When summed horizontally, these marginal utilities provide an aggregate marginal utility function in the sense of being the derivative of its own integral. The integral[15] for each good can be written as the concave function $U_i(Q_i)$ and $u_i(q_i)$, which serve as the Gossen-like utilities for America and England respectively. Lest anyone raise an eyebrow that I work with a collective concept such as total utility of a country, let me hasten to point out that this is completely rigorous because of my assumption that each person has labor-leisure of constant marginal utility. So long as workers never run out of leisure and provided that they have run into work, all is completely rigorous.

I shall leave to the reader the parallel case of England and shall concentrate on America. Our programming problem is to find the quantities that America is to produce of wheat and cloth, Y_1 and Y_2, so as to maximize national utility. If we select utility units so that one unit of labor corresponds to one util, and if we eliminate labor by substituting the production-possibility frontier of Ricardo, $L = A_1Y_1 + A_2Y_2$, we get the simple Kuhn-Tucker concave-programming problem

$$\underset{Y_i}{\text{Max}} \; U_1(Y_1-E_1) + U_2(Y_2) - A_1Y_1 - A_2Y_2 = U^*(-E_1,-E_2) \;, \quad Y_i \overset{\geq}{=} 0$$

where the levels of goods imported or exchanged, $-E_i$, are taken as given.

The well-known necessary and sufficient conditions for an optimal solution are given by

$$U_i'(Y_i-E_i) - A_i \overset{<}{=} 0 \text{ , with the inequality holding only when } Y_i = 0$$

This gives us as many conditions as there are goods. After we have substituted these optimal decisions into the maximand, we get the desired U* function that provides our generalized-Meade trade-indifference contours. The slopes of these U* contours are given by the ratios of the partial derivatives of the U function. And it is now evident that these slopes are the constant A_2/A_1 where both goods are being produced. Where America is producing wheat alone, our interesting case, the slope is given by $U_2'(-E_2)/A_1$ and the contour is strongly convex. The convex field joins up with the parallel-straight line field at a critical value of $-E_2^*$, and hence the boundary is the vertical line through the point market B in Figure 4. To the right of that vertical boundary, the Engel's curves are also vertical lines for America.

By similar mathematical reasoning, the reader will establish that Europe's critical boundary, at which her production of wheat ceases, is the horizontal line through the point C; and her Engel's curves in the interesting region below that boundary all have Engel's curves that are horizontal. And so, utilizing the income-propensities of my 1952 paper, we have the asymmetry needed to justify the orthodox position. Q.E.D.

Any reader who wishes to skip this mathematical argument is invited to compare the "contract locus" ABCD of Figure 4 with the CC contract locus of Figures 1 and 2 of my 1952 paper. The techniques developed there, when applied to Figure 4 will indeed vindicate the orthodox Keynes-Taussig presumption.

XI. Conclusion

We have come a long way. The orthodox way of thinking has been in a sense traced down to its possible intuitive origins. But to understand all is not to forgive all. The strong result of this paper has been generated--let us emphasize--by the assumption needed to make partial equilibrium invulnerable to the criticisms of those (like Viner) who have pointed out the lack of invariance of the supply and demand curves used in such analysis. The price for this defense of partial equilibrium is a high one--namely income effects that go completely on labor-leisure and not at all on goods. Little wonder then that the Ohlin argument cannot get off the ground, and orthodoxy is unscathed.

I hold no brief for orthodoxy. Actually my own vested interests are in the opposite direction. But I wonder whether we could not find asymmetric income effects traceable to a realistic substitutability of domestic factors and goods for export goods. And that might militate against the Ohlin agnostic position.

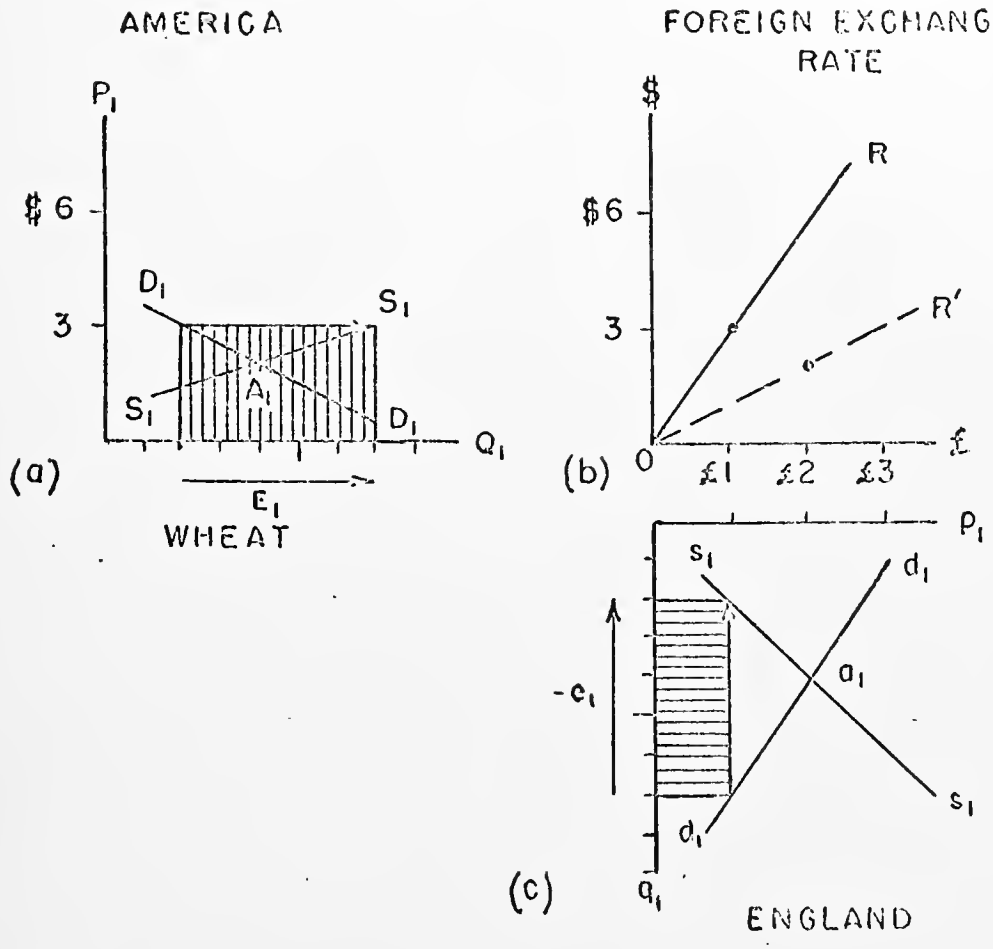

Figure 1. The OR ray in (b) translates each dollar P into its equivalent pound p. Wheat market equilibrium occurs where America's export arrow is just matched by England's import arrow. If wheat were the only good, America at OR would enjoy an export surplus measured in dollars by the rectangle in (a) or in pounds by the rectangle in (c). For 1-good balance of payments equilibrium, OR would have to depreciate to OR' so that both rectangles vanish. Any such clockwise move of the ray increases America's exports algebraically and reduces England's algebraic exports. Lowering R lowers all £p's and raises all $P's. Thus, in a 1-good case, an indemnity from America to England moves OR' counterclockwise to OR and induces a necessary depreciation of the payer's currency.

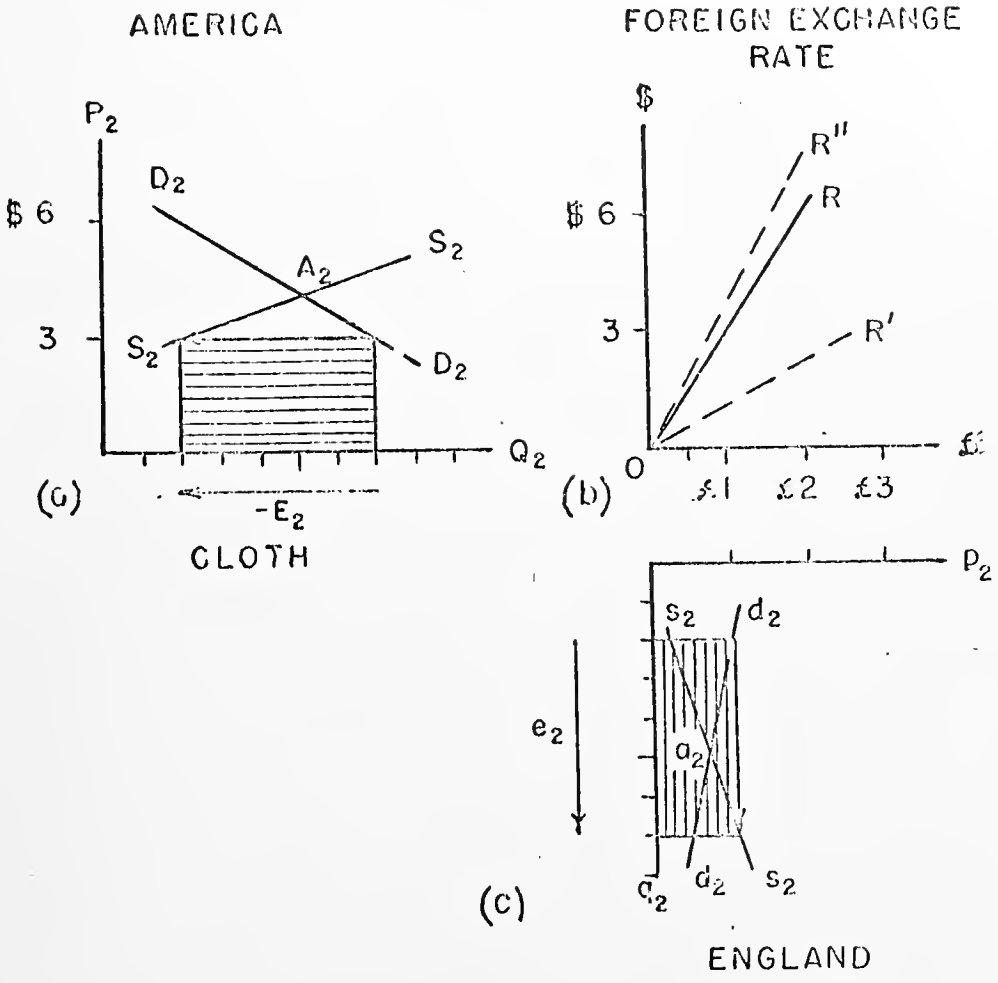

Figure 2. At each exchange rate, cloth equilibrium is found where arrows match as in Figure 1. To find 2-good balance of payments equilibrium, pivot ray until in either country export rectangle[s] cancel out import rectangle[s], as at OR. If now America pays transfer to England, OR must shift up to OR'' and currency of the payer depreciates. Because of Robinson curiosum, such a rise in R could perversely improve payer's terms of trade $P_1/P_2 = p_1/p_2$ but in Ricardian case where SS curves are horizontal and exchange market stable, payer's terms of trade must depreciate.

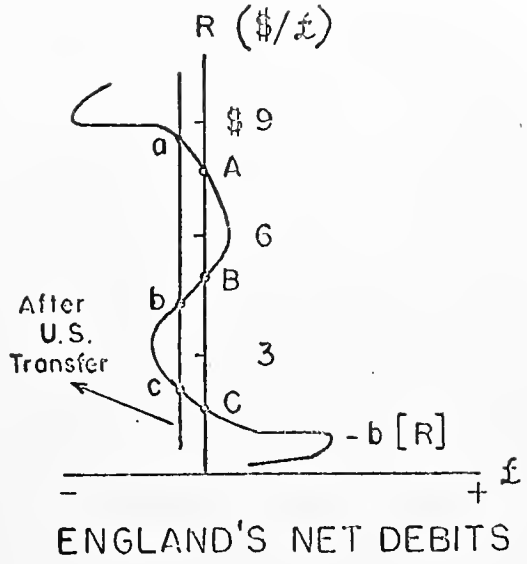

Figure 3. Here is a typical case where elasticity-pessimism does create unstable equilibrium B between stable equilibria A and C. Payment of transfer by America to England of t pounds is like shift of vertical axis leftward by amount t. New intersections give effect of transfer on exchange rate (and in constant cost case, equivalent deterioration of terms of trade). That stable equilibria shift upward to a and c is obvious. But conclusion that unstable B shifts to new unstable b, thereby perversely improving terms of trade of payer is quite false. In short run transfer produces positive excess demand at such excess demand R is dynamically bid up; final equilibrium end up at c. Thus, both comparatively statically as well as dynamically, payment depresses currency of payer--even though the unstable branch has opposite slope. The correspondence principal in the large is thus established for this one-dimensional case.

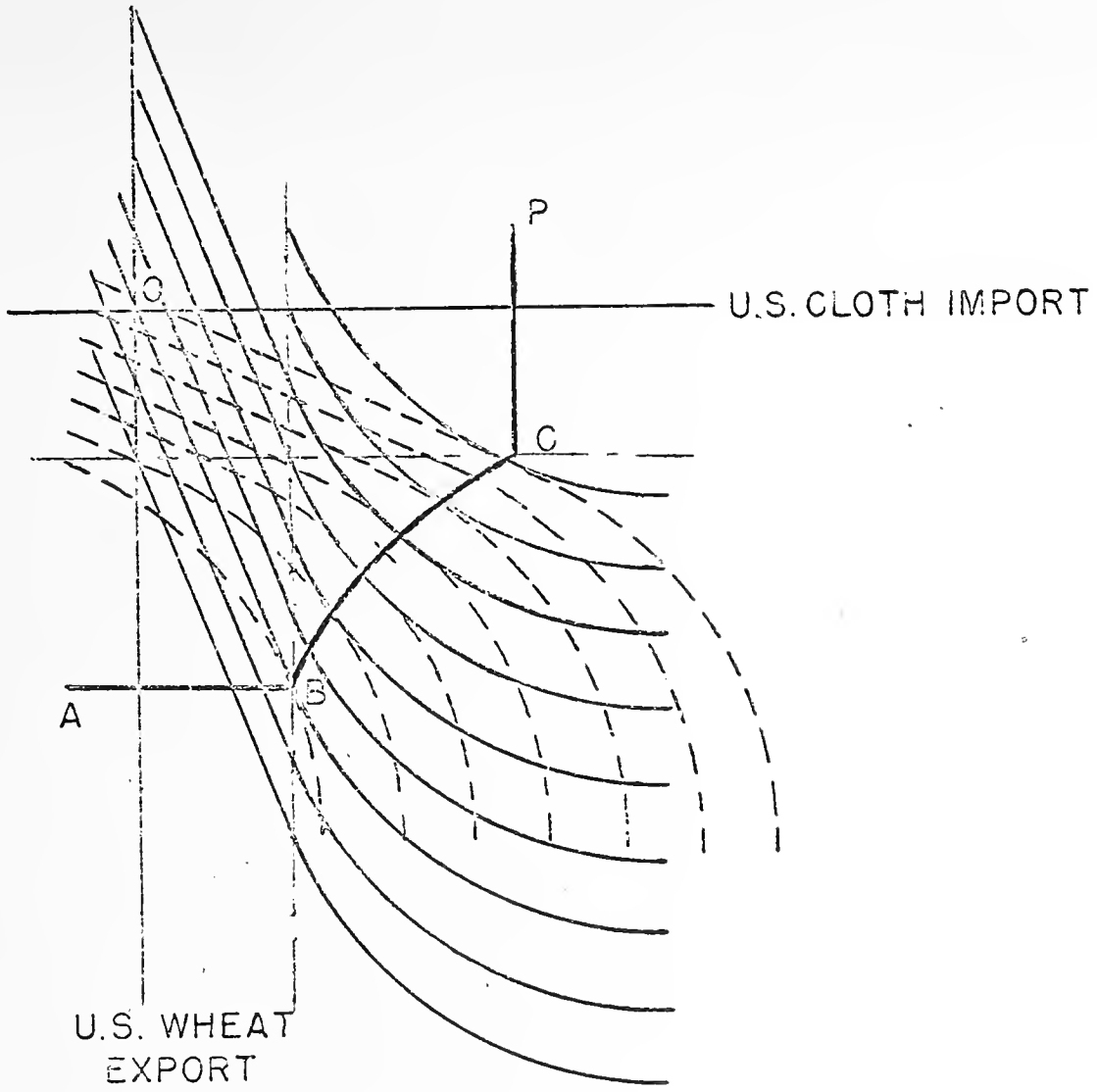

Figure 4. Reading upward and in the usual left-ro-right fashion, we see the American trade-indifference contours. Reading upside down and from right-to-left we see the English trade-indifference contours. Directly below the origin, both sets of contours are straight lines determined by comparative cost ratios. In case where America exports wheat and imports cloth, transfer moves equilibrium on the contract curve CB toward B, necessarily depreciating wheat/cloth price ratio.

FOOTNOTES

*Grateful acknowledgement is made to the National Science Foundation for research support and to Karen H. Johnson for assistance in the preparation of this paper.

[1]See for example the debate and rejoinders of J. M. Keynes and B. Ohlin, Economic Journal, Vol. XXXIX (1929), pp. 1-7, 172-182, 400-408. The principal article of each man is reproduced in H. S. Ellis and L. A. Metzler, eds. for the American Economic Association, Readings in the Theory of International Trade (Philadelphia: Blakiston Company, 1950), pp. 161-178. For a broad review see J. Viner, Studies in the Theory of International Trade (New York: Harper & Brothers, 1937), pp. 326-60. In the Taussig festschrift, Explorations in Economics, (McGraw-Hill, New York, 1936), W. W. Leontief provided an alleged possible example in which the secondary burden was on the receiver, and so strongly as to make the payer better off in consequence of its paying a transfer.

[2]A. C. Pigou, "The Effects of Reparations on the Ratio of International Exchange," Economic Journal, Vol. XLII (1932), pp. 532-42. This is summarized in A. C. Pigou, A Study in Public Finance, 3rd edition (London: MacMillan & Co., 1947), Chapter XIX.

[3]P. A. Samuelson, "The Transfer Problem and Transport Costs: The Terms of Trade When Impediments Are Absent," Economic Journal, Vol. LXII (June 1952), pp. 278-304, and P. A. Samuelson, "The Transfer Problem and Transport Costs, II: Analysis of Effects of Trade Impediments," Economic Journal, Vol. LXIV (June 1954), pp. 264-289. Both articles are reproduced in The Collected Scientific Papers of Paul A. Samuelson, Joseph E. Stiglitz, ed., (Cambridge, Massachusetts: M. I. T. Press, 1966), pp. 985-1037. In abridged form these are reproduced

in R. E. Caves and H. G. Johnson, eds. for the American Economic Association, Readings in International Economics (Richard D. Irwin, Inc., 1968), pp. 115-147.

[4]R. Jones, "The Transfer Problem Reconsidered," forthcoming in Economica.

[5]"Ricardo and Hume in an Exact, But Simple, General Equilibrium Model," to be published in the Journal of International Economics.

[6]As applied to single commodities in international trade, the familiar back-to-back diagram or its equivalent goes back to A. A. Cournot, Mathematical Principles of the Theory of Wealth (1838) tr. by Nathaniel T. Bacon (New York, Macmillan & Co., 1927),Chapter X; J. Viner, op. cit., pp. 589-91 gives references to Cunyngham (1904), Barone (1908), Pigou (1904), and H. Schultz (1935); also see C. F. Bickerdike, Economic Journal, XVII (1907), p. 98 for simple Cournot-like formulas; for a non-graphic literary exposition see F. W. Taussig, Some Aspects of the Tariff Question (Cambridge, Mass: Harvard University Press, 1915 and 1931), Chapter I; see also G. Haberler, The Theory of International Trade (London: William Hodge & Co., Ltd., 1936), Chapter 15, pp. 227-236, and also the reference there to R. Schüller (1905); or see P. A. Samuelson, Economics, 8th edition (New York; McGraw-Hill Book Company, 1970), Chapter 34, pp. 651-653, whose Figure 34-2 has identical functions to those of Figure 1 here. The 1967, 1964 and 1958 editions also contain similar diagrams in Part Five.

The Cournot problem has been generalized to any number of commodities

and given a solution by S. Enke in "Equilibrium Among Spatially Separated Markets: Solution by Electric Analogue," Econometrica, Vol. XIX (January, 1951), pp. 40-47; see also P. A. Samuelson, "Spatial Price Equilibrium and Linear Programming," American Economic Review, Vol XLII, No. 3 (June 1952), pp. 283-303 and P. A. Samuelson, "Intertemporal Price Equilibrium: A Prologue to the Theory of Speculation," Weltwirtschaftliches Archiv, Band 79, Weft 2 (Hamburg: Hoffman & Campe Verlag, December 1957), pp. 181-219; both these articles are reproduced in The Collected Scientific Papers of P. A. Samuelson, op. cit., Vol II, pp. 925-984. Karl Fox and also G. C. Judge and T. Takayama have generalized the problem to multiple commodities.

As applied to exchange rate equilibrium, the earliest exact reference seems to be C. F. Bickerdike, "The Instability of Foreign Exchange," Economic Journal, Vol. XXX (1920), pp. 118-22 (which in sense pre-dates Pigou's 1932 rigor); J. Robinson, Essays in the Theory of Employment (London and New York: Macmillan & Co., 1937), pp. 183-228 further advances the subject. For graphical formulation and advance see G. Haberler, "The Market for Foreign Exchange and the Stability of the Balance of Payments: A Theoretical Analysis," Kyklos, Vol. III (1949), pp. 193-218 which is reproduced in International Finance, Selected Readings, R. N. Cooper, ed. (Harmondsworth, England: Penguin Books, Ltd., 1969), pp. 107-134, also G. Haberler, "Currency Depreciation and the Terms of Trade," in Wirtschaftliche Entwicklung und soziale Ordnung, Ernst Lagler and Johannes Messner, eds. (Vienna: Verlag Herold, 1952), pp. 149-158. In the modern literature innumerable writers have worked out elasticity expressions for so-called Marshall-Lerner

stability conditions based upon such models. Similar matters are ingeniously formulated in John Burr Williams, International Trade Under Flexible Exchange Rates (Amsterdam: North-Holland Publishing Co., 1954). See also T. O. Yntema, A Mathematical Reformulation of the General Theory of International Trade (Chicago, Ill.: the University of Chicago Press, 1932), a beautiful work that has never been appreciated at its true worth.

[7]The usual perverse violation of Marshall-Lerner stability conditions is not possible at a one-good autarky point.

[8]It is well known that great inelasticity of the functions could negate the Marshall-Lerner conditions for such stability. Provided the curves have the usual shapes, there will exist at least one finite equilibrium R* at which trade balances. Our drawing satisfies stability conditions. And generally the Ricardo constant-cost case, which we shall discuss later, makes for high elasticity and a tendency toward stability.

[9]This result does not have to be qualified even for an unstable equilibrium as I shall demonstrate in Figure 3 below. If fixed parities are imposed by the gold standard, the Hume mechanism accomplishes the equivalent result by ultimately lowering the W in the P_i/W expressions and raising the w in the p_i/w expressions so that w/W appreciates by exactly the same percentage as does the exchange rate R in my exposition. Part of Keynes's skepticism concerning the feasibility of transfer had nought to do with the change in the terms of trade as such as with his skepticism concerning the ease with which the conventional gold standard mechanism made adjustments under a re-

gime of fixed parities. Perhaps he would have been less skeptical if he had been contemplating a regime of floating exchange rates. In the unpublished paper already referred to, I prove the illogic of those who oppose floating exchange rates on the grounds of elasticity-pessimism, demonstrating that if the econometric conclusions of these critics is correct, then the equilibrium under the Hume mechanism is also unstable.

[10]J. Robinson, op. cit., pp. 218-221, particularly p. 219 n.1.

[11]P. A. Samuelson, "Theoretical Notes on Trade Problems," Review of Economics and Statistics, Vol. XLVI, No. 2 (May 1964), pp. 145-154, reproduced in The Collected Scientific Papers of Paul A. Samuelson, op. cit., Vol. II, pp. 821-830.

[12]For the heuristic "correspondence principle," see P. A. Samuelson, Foundations of Economic Analysis (Harvard Press, Cambridge, Mass., 1947, and in paperback, Atheneum, New York, N.Y., 1965), Chapters IX, X and Appendixes A, B. As applied to a linear system or to the linear approximation near equilibrium of a non-linear system -- say to $\Delta y_i = A[y_i] + [b_i]$, or to $d/dt[y_i]A[y_i]+[b_i]$--its comparative statics will depend on the non-singularity of its Jacobian matrix A, while its dynamic properties will depend on the latent roots of A defined by $\det[A-(1 + x)I] = 0$, or by $\det[A-xI] = 0$. When a root, x_1, becomes or passes through zero, the qualitative dynamic behavior reverses (as from stability to instability) and so too will the comparative static behavior (as from normality to perverseness). As an example, when $k = 1 + x$ in the multiplier expression $1 + k + k^2 + \ldots$ passes through

unity and above, dynamic instability occurs; at the same point, the comparative static multiplier expression, $1/(1 - k)$ perversely turns negative. But, as will be argued in connection with Figure 3 below, $1/(1 - k)$ is not really a true comparative static observed phenomenon.

Since 1947, a number of economists have questioned the generality and unambiguity of the correspondence principle, so let me make some disclaimers here. When x_1 is complex rather than real and has its real part change sign as x_1 passes through a pure imaginary number or has $|1 + p|$ pass through unity, the argument needs modification since only the dynamic behavior is reversed. Furthermore, the heuristic principle works best when A is a definite matrix as in connection with an extremum problem; for then real roots are assured, and not only do we have knowledge about A but also about its principal minors. Similarly if A is a Leontief-Metzler-Frobenius matrix, in which all off-diagonal elements are of one sign and A possesses diagonal dominance, then similar sweeping conclusions are possible about principal minors and the reality of the relevant root; and indeed, as I pointed out in the 1960 Frisch festschrift, my version of the LeChatelier principle holds for such structures. See p. A. Samuelson, "An Extension of the LeChatelier Principle," Econometrica 28 (1960), pp. 368-379 (reproduced, with the first paragraph misprint "inelastic" corrected to read "elastic," in Collected Scientific Papers I, pp. 626-637).

[13]See P. A. Samuelson, Economic Journal (1954), op. cit. The only definite conclusion reached there was that the orthodox presumption had much to be said for it; but where the impediment involved real, exhaustive cost, Ohlin agnosticism seemed justified.

[14]Of course the present theorem is a restatement, from a different point of view, namely the 1952-54 viewpoint, of what has already been established in this paper.

[15]Except for the algebraic sign of the import variable, these are what I introduced as "social-payoff" functions in the cited articles on spatial and intertemporal price equilibrium: P. A. Samuelson, American Economic Review (1952), op. cit., pp. 283-303 and Weltwirtschafliches Arkiv (1957) op. cit., pp. 181-219. It may be asked, how are collective indifference curves possible in view of my earlier proof that they exist only when the indifference curves are homothetic and identical between citizens? The present case does not quite fall into that category. But, as has been discussed elsewhere, there is a wider class in which collective indifference contours hold within a range, namely the Gorman-Theil case where all Engel's curves are straight lines with slopes (at each price ratio) common to all the citizens. Now actually the only lines that can stay straight lines through the non-negative orthant are lines through the origin -- my homothetic case. In the present example, where the Engel's curves are lines parallel to the leisure axis, when the individual stops buying leisure, because he cannot afford any, or when he buys 24 hours of leisure because it does not pay him to work at all, we lose the simplifications that constancy of marginal utility does for us in providing solid underpinning for partial equilibrium. So, as in all Gorman-Theil deviations from my narrower conditions, the collective indifference curves will break down as a concept in the large. (Actually,

for indemnity large enough, some Englishmen will quit working and some American will find himself working every possible hour. And then partial equilibrium cannot be used.) However, for the purpose of the present exercise, within a range of indemnities, the trick does work even though it cannot work unconditionally.

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